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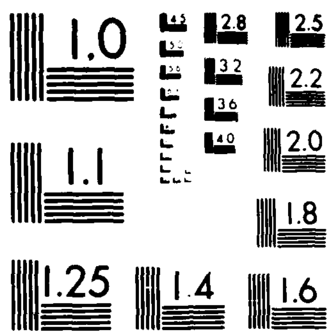

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This is the final report of a project carried out in the Applied Mathematics Groups of the Department of Mathematics at Stanford University. Results were obtained on the stability of nonlinear waves and of solutions of nonlinear amplitude equations of the Ginzburg-Landau type. New results were obtained on solutions of the Kortweg-de Vries equation. Uniform solutions for scattering of waves by a potential barrier were constructed. The stability regions for Hill's equation were determined, which can be used to describe waves in periodic media.

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MATHEMATICAL PROBLEMS OF NONLINEAR WAVE PROPAGATION AND OF WAVES IN HETEROGENEOUS MEDIA

FINAL REPORT

October 1, 1984 - September 30, 1986

Professor Joseph B. Keller

October 22, 1986

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

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I. BRIEF OUTLINE OF RESEARCH FINDINGS

Most of our findings during the last years of research are contained in the research papers listed in Section II. Some of them have been published already, others have been submitted for publication and accepted, and others have not yet been accepted. The status of each paper is indicated after its title. In addition in Section III we give abstracts of papers submitted during this period. Now we shall mention some of the findings explicitly.

Concerning nonlinear wave propagation, Dr. Newton and Prof. Keller have analyzed the stability of a large class of nonlinear waves, and Prof. Keller has derived amplitude equations for resonantly interacting water waves. Dr. Newton studied instabilities of solutions of amplitude equations of the Ginzburg-Landau type by examining secondary bifurcation. Professor Venakides has completed two papers on the Korteweg-de Vries equation. One shows how step-like initial data produce a train of solitons by continually emitting them one at a time. The other shows how oscillatory waves are produced from smooth initial data in the weak dispersion limit. This latter paper fills a gap in the theory of wave production, and should ultimately enable one to connect Whitham's modulation theory to initial non-oscillatory data.

Another work by Professor Keller provides uniform solutions for scattering of waves by a potential barrier. This well known problem is not usually solved by uniform methods in the literature. In particular the phases of the reflection and transmission coefficients are not available. These quantities are needed to calculate the discriminant of Hill's equation containing a periodic distribution of barriers. This in turn can be used to examine the linear stability of nonlinear waves.

In the theory of waves in heterogeneous media, Dr. Weinstein and Professor Keller have written two papers on the stability regions for Hill's equation, which governs waves in periodic media. Mr. Nevard and Professor Keller have proved a reciprocal theorem and an inequality for the effective conductivities of heterogeneous anisotropic media. It generalizes the previous theorems on the topic, and can be used to treat low frequency waves.

93.	A. Spence A. Jepson	The numerical computation of turning points of nonlinear equations Pub: Treatment of Integral Equations by Numerical Methods, 169-183, London, 1982.
94.	J. B. Keller R. Burridge	Biot's poroelasticity equations by homogenization Pub: Springer Lecture Notes, in Macroscopic Properties of Disordered Media, NY, 51-57,1982.
95.	J. B. Keller	Capillary waves on a vertical jet Pub: J. Fluid Mech., 135, 171-173, 1983.
96.	J. B. Keller A. S. Whittemore	Survival Estimations Using Splines Pub: Biometrics, 42, 495-506, 1986.
97.	J. B. Keller J. F. Geer	Eigenvalues of slender cavities and waves in slender tubes Pub: J. Acoust. Soc. Am., 74, 1895-1904, 1983.
98.	J. B. Keller R. Voronka	Valuation of stocks and options To be submitted.
99.	M. Cheney	Inverse scattering in dimension two Pub: J. Math. Phys., 25 (1), 94-107, 1984.
100.	K. C. Nunan J. B. Keller	Effective viscosity of a periodic suspension Pub: J. Fluid Mech., 142, 269-287, 1984.
101.	K. C. Nunan J. B. Keller	Effective elasticity Tensor of a Periodic Composite Pub: J. Mech. Phys. Solids, 32, 259-280, 1984.
102.	J. B. Keller	Breaking of liquid films and threads Pub: Phys. Fluids, 26, 3451-3453, 1983.
103.	J. B. Keller G. R. Verma	Hanging rope of minimum elongation Pub: SIAM Rev., 26, 569-571, 1984.

104.	M. Cheney S. Coen A. Weglein	Velocity & density of a two-dimensional acoustic medium from point source surface data Pub: J. Math. Phys., 25, 1857-1861, 1984.
105.	J. B. Keller	Probability of a shutout in racquetball Pub: SIAM Rev. , 26, 267-268, 1984.
106.	S. Venakides	The Zero Dispersion Limit of the Korteweg-de Vries Equation for Initial Potentials with Non-trivial Reflection Coefficient Pub: Comm. Pure Appl. Math. 38, 125-155, 1985.
107.	J. H. Maddocks	Stability of Nonlinearly Elastic Rods Pub: Arch. Rat. Mech. Anal., 85, 311-354, 1984.
108.	J. B. Keller	Genetic Variability Due to Geographical Inhomogeneity Pub: J. Math. Biol., 20, 223-230, 1984.
109.	J. B. Keller M. S. Falkovitz	Precipitation pattern formation In preparation.
110.	V. Twersky	Scattering and Nonscattering Obstacles Pub: SIAM J. Appl. Math., 43, No 4, 1983.
111.	R.E. Caflisch J.H. Maddocks	Nonlinear Dynamical Theory of the Elastica Pub: Proc. Roy. Soc. Edin., 99A, 1-23, 1984.
112.	J.B. Keller J.G. Watson	Rough Surface Scattering via the Smoothing Method Pub: J. Acoust. Soc. Am., 75, 1705-1708, 1984.
113.	J.B. Keller	Free boundary problems in mechanics Pub: Lectures in Partial Differential Equations S.S. Chern, editor, Springer New York, 99-115, 198
114.	J.B. Keller	Newton's second law Sub: Am. J. Physics

115.	L.L. Bonilla	Effective elastic constants of polycrystalline aggregates Pub: J. Mech. and Phys. Solids, 33, 227-240, 1985.
116.	J. Fawcett	On the Stability of Inverse Scattering Problems Pub: Wave Motion 6, 489-499, 1984.
117.	J. Fawcett	Two Dimensional Modelling and Inversion of the Acoustic Wave Equation in Inhomogeneous Media Pub: Stanford Exploration Project, Dept. of Geophysics, Stanford University, 38, 297-313, May 1984.
118.	J. Fawcett H.B. Keller	Three Dimensional Ray Tracing and Geophysical Inversion in Layered Media Sub: SIAM J. Appl. Math.
119.	L.L. Bonilla J.B. Keller	Acousto-elastic effects and sound wave propagation in heterogeous anisotropic materials Pub: J. Mech. and Phys. Solids, 33, 241-261, 1985.
120.	J. Fawcett R.W. Clayton	Tomographic Reconstruction of Velocity Anomalies Sub: Bulletin of the Seismological Soc. Am.
121.	J. Fawcett	Inversion of N Dimensional Spherical Averages Pub: SIAM J. Appl. Math., 45, 336-341, 1985.
122.	M.I. Weinstein J.B. Keller	Hill's equation with a large potential Pub: SIAM J. Appl. Math., 45, 200-214, 1985.
123.	M. Cheney S. Coen A. Weglein	Inversion of the 2.5-D Acoustic Equation Pub: Proceedings of the Conference on Inverse Scattering Theory and Application, Tulsa, 1983.
124.	M. Cheney J.H. Rose B. DeFacio	The connection between time- and frequency- domain three-dimensional inverse scattering methods Acc: J. Math. Phys.
125.	M. Cheney	A rigorous derivation of the 'Miracle' of three-dimensional inverse scattering theory Pub: J. Math. Phys.,25, 2988-2990, 1984.
126.	E.O. Tuck	Small Gap Flows (A Lecture Series) Pub: Applied Mathematics Group, Dept. of Mathematics, Stanford University, AMG-84-126, March 84. and Dept. of Naval Architecture & Offshore Engineering, University of California, Berkeley, NAOE 84-1, April 1984.

127.	J.B. Keller	Discriminant, transmission coefficient and stability bands of Hill's equation Pub: J. Math. Phys. 25, 2903-2904, 1984.
128.	J.B. Keller	One hundred years of diffraction theory Pub: Trans. IEEE PGAP-33, 123-126, 1985.
129.	J.B. Keller	Soliton generation and nonlinear wave propagation Pub: Phil. Trans. R. Soc. Lond. A 315, 367-377, 1985.
130.	J.B. Keller	Semiclassical Mechanics Pub: SIAM Review 27, 485-504, 1985.
131.	S. Venakides	The generation of modulated wavetrains in the solution of the KdV equation Pub: Comm. Pure Appl. Math. 38, 883-909, 1985.
132.	S. Venakides	Long-time asymptotics of the KdV equation Acc: AMS Transactions
133.	J.B. Keller	Computers and chaos in mechanics Pub: in Theoretical and Applied Mechanics, F.I. Nioordson and N. Olhoff, eds., Elsevier, 31-41, 1985.
134.	J.B. Keller J. Nevard	Reciprocal relations for effective conductivities of anisotropic media Pub: J. Math. Phys. 26, 2761-2765, 1985.
135.	J.B. Keller Luis Bonilla	Irreversibility and nonrecurrence Pub: J. Statistical Physics 42,1115-1125, 1986.
136.	J.B. Keller W. Boyse	Inverse elastic scattering in three dimensions Pub: J.A.S.A. 79, 215-218, 1986
137.	J.B. Keller	Reaction kinetics on a lattice Pub: J. Chem Phys. 84, 4108-4109, 1986
138.	J.B. Keller	Impact with friction Pub: ASME J. Appl. Mech 53, 1-4, 1986
139.	J.B. Keller	Uniform Solutions for scattering by a potential well. Pub: AM. J. Phys., 54, 546-550, 1986

140.	J.B. Keller	The Probability of heads Pub: Am. Math. Monthly 93, 191-197, 1986.
141.	J.B. Keller	Melting and freezing at constant speed Pub: Physics Fluids, 29, 2013, 1986.
142.	J.B. Keller	"Acoustoelasticity" Pub: in Amorphorous Polymers and Newton Fluids, J. Ericksen, D. Kinderleherer, S. Prager and M. Tirrell, eds., Springer, Berlin, 1987.
143.	J.B. Keller	Effective conductivities of reciprocal media Acc: Random Media, G. Papanicolaou, ed. Springer, Berlin, 1986.
144.	J.B. Keller	Impact with an impulsive frictional moment Acc: ASME J. Appl. Mech.
145.	J.B. Keller J. Maddocks	Ropes in Equilibrium Acc: SIAM J. Appl. Math
146.	J.B. Keller J.M. Vanden-Broeck	Weir Flows Acc: J. Fluid Mechanics
147.	Walter Craig	An existence theory for water-waves and the Boussinesq and Kortweg-deVries Scaling limits Pub: C.P.D.E. 10 (8), 787-1003, 1985.
148.	Walter Craig	The Lyapunov index and the integrated density of states for Stochastic Schrödinger operations Pub: Infinite dimensional analysis and Stochastic Process 45, 16-27, 1986.
149.	P.K. Newton L.K. Sirovich	Instabilities of the Ginzburg-Landau equation: Periodic solutions Pub: Quarterly of Applied Mathematics, 49-58, 1986.
150.	P.K. Newton L.K. Sirovich	Periodic Solutions of the Ginzburg-Landau equation Pub: Physica 21D, 115-125, 1986.

151. P.K. Newton "Ginzberg-Landau equation: Stability and Bifurcations," in Stability of Time Dependent and spatially varying flows, Y. Houssani and D.L. Dwoyer, eds., Springe: Berlin, 1986.

III. ABSTRACTS OF MANUSCRIPTS SUBMITTED DURING REPORT PERIOD

Reciprocal relations for effective conductivities of anisotropic media

by: J. Nevard and J. B. Keller

We consider any pair of two-dimensional anisotropic media with local conductivity tensors which are functions of position and which are related to one another in a certain reciprocal way. We prove that their effective conductivity tensors are related to each other in the same way for both spatially periodic media and statistically stationary random media. We also prove an inequality involving the effective conductivity tensors of two three-dimensional media which are reciprocally related. These results extend the corresponding results for locally isotropic media obtained by Keller, Mendelsohn, Hansen, Schulgasser and Kohler and Papanicolau. They also yield a relation satisfied by the effective conductivity tensor of a medium reciprocal to a translated or rotated copy of itself.

Uniform solutions for scattering by a potential barrier

by: J. B. Keller

The one dimensional Schrödinger equation is solved asymptotically for scattering of a particle by a potential barrier and for bound states of a potential well, when the potentials change little in a wavelength. Both solutions are represented uniformly in space, rather than nonuniformly as in the WKB method. This avoids matching expansions and using connection formulas. The scattering solution and the complex reflection and transmission coefficients are also uniform in the particle energy.

Irreversibility and nonrecurrence

by: L. L. Bonilla and J. B. Keller

Can the irreversible, nonrecurrent equations of macroscopic physics be derived exactly from the reversible recurrent equations of classical mechanics? We show by an example that it is possible to derive an irreversible equation from reversible ones exactly, with no approximations. However the resulting equation has a damping coefficient which can have any value, positive or negative, depending upon the initial conditions. By choosing the initial conditions in a particular way, we derive the Langevin equation with an external force satisfying the fluctuation-dissipation theorem. We then describe the general projection method for deriving irreversible equations, and some of its applications. We also show, in an example, how to get nonrecurrent reversible equations from recurrent reversible ones, by letting the number N of degrees of freedom become infinite. For this example, we compare the solutions of the equations with various finite values of N with those for $N = \infty$ to show how long they are close to one another.

Long-time asymptotics of the KdV equation

by: S. Venakides

We study the long time evolution of the solution to the Korteweg-de Vries equation with initial data $v(x)$ which satisfy:

$$\lim_{x \to -\infty} v(x) = -1 \qquad \lim_{x \to +\infty} v(x) = 0$$

We show that as $t \to \infty$ the step emits a wavetrain of solitons which asymptotically have twice the amplitude of the initial step. We derive a lower bound on the number of solitons generated up to time t for t large.

The Generation of modulated wavetrains in the solution of the KdV equation

by: S. Venakides

We study the solution $u(x,t,\epsilon)$ of the initial value problem for the Korteweg-de Vries equation:

$$u_t - 6uu_x + \epsilon^2 u_{xxx} = 0 \qquad u(x,0,\epsilon) = v(x)$$

where $v(x) \leq 0$ is a single well. We introduce a method which can be generally applied to the solution of completely integrable systems in the continuum limit of the spectral data. We recover the weak limit of $u(x,t,\epsilon)$ as $\epsilon \to 0$, computed earlier by Lax and Levermore. Furthermore, we show the mechanism by which fast oscillations emerge in regions of the x-t plane, called shock regions, and we describe the nature of these oscillations.

REACTION KINETICS ON A LATTICE

by: Joseph B. Keller

The kinetics of an irreversible first order reaction at the points of a lattice is examined. A "mean field" approximation is used.

WEIR FLOWS

by: Joseph B. Keller and Jean-Marc Vanden-Broeck

The flow of a liquid with a free surface over a weir in a channel is calculated numerically for thin weirs in channels of various depths, and for broad crested weirs in channels of infinite depth. The results show that the upstream velocity, as well as the entire flow, are determined by the height of the free surface far upstream and by the geometry of the weir and channel, in agreement with observation. The discharge coefficient is computed for a thin weir, and a formula for it is given which applies when the height of the weir is large compared to the height of the upstream free surface above the top of the weir. The coefficients in this formula are close to those found empirically.

SEMICLASSICAL MECHANICS

by Joseph B. Keller

Classical mechanics and the quantum conditions of Planck, Bohr, Sommerfeld, Wilson and Einstein are presented. The virtues and defects of this "old quantum theory" are pointed out. Its replacement by quantum mechanics is described leading to the Schrödinger equation for the wave function and the corresponding energy eigenvalues. For separable systems, the reduction of this equation to ordinary differential equations and their asymptotic solution by WKB method are described, as well as the resulting corrected quantum conditions with integer or half-integer quantum numbers. For nonseparable systems, the analogous asymptotic solution constructed by the author is described, together with the corrected quantum conditions to which it leads. Examples of the use of these conditions in the solution of eigenvalue problems are presented. It is explained that difficulties arise in using this method when the classical motion is stochastic or chaotic. Suggestions for overcoming these difficulties are mentioned.

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